Version of the March 19, 2021

SPECIALITY : MATH. MODELING & AI, SYLLABUS OF SCIENTIFIC COURSES

Academic Year: 2021-2022

4TH YEAR

1ST SEMESTER

SIGNAL 1

Hilbert spaces: Linear forms; prehilbertian spaces, Hilbert spaces; Projection on convex spaces; Hilbertian bases; Examples (Fourier, orthogonal polynomials).
Fourier Transform: Fourier decompositions of a periodic function; Fourier Transform of a function defined on R; Convolution; Discrete Fourier Transform; FFT algorithm.
Numerical signal and Image processing, compression, denoising. Examples of sound processing

OPTIMIZATION

Deterministic differentiable optimization: Existence and unicity of constrained optimization (including convexity); Tangent cone, Farkas lemma, KKT points; Line search, Wolfe conditions; First order algorithms for unconstrained optimization; Second order algorithms for unconstrained optimization (including BFGS), Lagrangian duality; First and second order algorithms for unconstrained optimization (projected gradient, SQP, penalization methods; interior points, augmented Lagrangian); Algorithm convergence
Discrete stochastic optimization: Metropolis-Hastings method for simulating, approximately, a given probability distribution known modulo a multiplicative constant (construction of a reversible Markov chain, convergence to the invariant measure, speed of convergence); Simulated annealing algorithm (Gibbs measure, temperature scheme, proof of the algorithm convergence, parameter adjustment in practice).

ELEMENT OF STATISTICAL MODELLING

Nonparametric statistics: empirical distribution function, Kolmogorov test, normality tests
Chi-square goodness-of-fit test, Chi-square independence test
Linear models: estimation of the parameters, confidence intervals, prediction intervals, Fisher test for a sub-model, model selection and model validation. Linear regression, ANOVA, ANCOVA
Generalized linear model: statistical inference, variable selection. Logistic regression, loglinear model.

QUALITY, SECURITY, ENVIRONMENT (QSE)

Quality approach: Statistical process control (SPC); notions of metrology; capability of a process; control cards; the SPC system in a company.
Safety: Notions of risks, evaluation, prevention, protection. Notions of IT security: bases of cryptography; electronic certificates; protocol https; digital signature.
Environmental protection: regulations, ISO14000.

HPC, MATRIX COMPUTATIONS AND LARGE SPARSE SYSTEMS

Eigenproblems : Eigenvalue problems, conditioning and Schur factorization.
Methods for eigenvalue problems.
Paradigmes and languages: Introduction to computer architecture (processing units and memory hierarchy macroscopic organization), Introduction to the different paradigms of parallel programing, Description of the OpenMP API, Description of the MPI run-time directives.
Sparse systems: Sparse storage techniques, Krylov subspace methods: Conjugate gradients, GMRES, Preconditioning techniques (stationary iterative methods, incomplete factorizations, sparse inverse approximates). Reordering algorithms for direct methods

INSA TOULOUSE

ELECTIVE COURSES: ONE AMONG TWO

1. ADVANCED PROBABILITY AND MONTE CARLO METHODS

Martingales: Conditional expectation, filtration, martingale, submartingale and supermartingale, Doob's theorem, optional stopping theorem, convergence theorems, law of large numbers and central limit theorem for martingales.

Stochastic Algorithms: Background on deterministic gradient descent, Introduction to Robbins-Monro algorithms and links with classical results (Law of Large Numbers), Robbins-Siegmund Lemma, Robbins-Monro Convergence Theorems, Applications (Two-Armed Bandit, quantile, quantization, Linear Regression in high dimension).

Monte Carlo Methods: Generation of random numbers, simulation by inversion of the distribution function, by the reject method and by some specific methods, Monte-Carlo Methods (convergence, rate of convergence, variance reduction by using different methods).

2. PARTIAL DERIVATIVE EQUATIONS AND MONTE CARLO METHODS

Linear PDE (analysis, numerics):

PDE models (examples, the four fundamental linear PDE. Classification. Principle of the Finite Difference method: consistency, stability, convergence. Laplace-Poisson equation (elliptic): explicit solution (by separation of variables), maximum principle, FD schemes. Heat equation (parabolic): explicit solution (by Fourier transform), FD schemes (explicit, implicit, splitting). A non-linear case. Transport equation (hyperbolic): explicit solution - characteristics, schemes, equivalent equation. Waves equation (hyperbolic): explicit solution - characteristics, schemes. Practical works: stability, accuracy; modeling.

Monte-Carlo:

Generation of random numbers, simulation by inversion of the distribution function, by the reject method and by some specific methods, Monte-Carlo Methods (convergence, rate of convergence, variance reduction by using different methods).

2ND SEMESTER

SIGNAL II AND OPTIMIZATION

Wavelets:

Wavelet transforms. Basis of wavelets: definition and properties. Multiresolution analysis. Discrete wavelets: Haar basis, filter banks with exact reconstruction, 1d bi-orthogonal wavelets, wavelet decomposition, spatial and frequency localization, 2d wavelets. Introduction to jpeg, denoising. Greedy algorithms such that Matching Pursuit and Orthogonal Matching Pursuit.

Algorithms for unconstrained nonsmooth optimization:

Elements for convex analysis: Notion of convex subdifferential, optimality necessary condition, Lagrangian duality. Algorithms: subgradient methods, proximal methods, forward-backward algorithm. Some examples: minimax problems, proximal algorithms, how to manage constraints.

PROJECT RESEARCH/INNOVATION

After a call of subjects from private sector (aeronautic, spatial, pharmaceutical sectors) or famous laboratories (LISBP, LAAS, IMFT, LPNO, CEMES…), groups of 2 or 3 students have to resolve mathematical modelling problems, supervised by specialists of the concerned domain (physics, mechanics, biology, finance, reliability…). The essential point is the realistic situation in front of a complex, new problem, whose solution is not known, even by the supervisor.

MACHINE LEARNING

Introduction to machine learning. Optimization of the bias / variance trade-off. Model selection via penalized criterion: Mallows CP, BIC, Ridge, Lasso… Linear and quadratic discriminant analysis, k nearest neighbors. Clustering with k-means. Classification and regression trees. Bagging, random forests. Neural networks, multilayer perceptron, backpropagation algorithms, optimization algorithms, introduction to deep learning. Missing data imputation. Scientific deontology and statistical decision. Legal framework and societal impacts of AI

ELECTIVE COURSES: TWO AMONG FOUR

1. FINITE ELEMENT METHODS & MODEL REDUCTIONS

Analysis of elliptic PDEs: weak solution vs classical solution, Sobolev spaces, Lax-Milgram theory, a-priori estimations. Boundary conditions. Energy minimization.Modelling with a FE software (eg FreeFEM++) : classical models, 1 real-like problem.
Finite Element Method principles: discretisation, approximation, data structures, implementation. Error analysis (a-priori). Convergence curves, assessment of computational codes. Transport term, stabilisation (SUPG, characteristics). Unsteady models: semi-discretisation. Non-linear models & linearization.
Weak constraints: mix formulations, Lagrangian. Dirichlet B.C. u=g and u.n=g. (Navier-)Stokes model : discrete inf-sup condition, FE schemes. Coupling Mortar, see « Structural mech. » course
Practical, Programming (Python): assembly algorithm ; code assessments, scheme accuracy, error estimator.
Domain Decomposition Method (DDM): an introduction

2. MODELING AND SCIENTIFIC COMPUTING IN FLUID AND STRUCTURAL MECHANICS

Brief review of the general concepts in continuum mechanics
Modeling and scientific computing in fluid mechanics: Dynamics of inviscid fluids ; Dynamics of Newtonian viscous fluids ; Capillary phenomena ; Numerical solution of fluid dynamics equations with the finite volume method (FVM) ; Implementation of the FVM to solve a model problem (dam break).
Modeling and scientific computing in structural mechanics: Variational formulation and mathematical analysis of the elasticity problem ; Numerical resolution of elasticity with the finite element method ; Multiscale model and code coupling ; Application: modeling and computation of static as well as dynamic elastic problems through the use of an industrial software + development of python codes for the computation of stress concentration and local propagation of cracks within solids.

3. STOCHASTIC PROCESSES: TIME SERIES AND GAUSSIAN PROCESSES

Time series. Introduction and Descriptive Analysis (Time series decomposition, Estimation and Elimination of Trend and Seasonal Components), Random Modeling of Time Series (Stochastic process, stationnarity, Autocovariance Function), Statistical Inference of Stationary processes of order 2 (Moment Estimation, Best linear predictor, Partial autocorrelation, statistical tests), ARMA and ARIMA Models (AR process, MA process, ARMA et ARIMA processes).
Gaussian Processes. Introduction to real-valued Gaussian processes in discrete time; extension to the continuous case; parametric estimation using discrete martingale tools. On the importance of the covariance function: spectral aspects and link with the regularity of the process. Simulation of Gaussian processes and its use in modeling, Application to real data in Geostatistic (meteorology).

4. DATA ANALYSIS

Introduction to exploratory analysis of big data, Syntax and objects of R and Python languages, functions, object and functional programming (python). Multivariate exploratory statistical analysis. Principal component, discriminant and correspondence analyses, multidimensional scaling, NMF, hierarchical clustering, k-means, mixture models and DBSCAN.

5TH YEAR

1ST SEMESTER

RESEARCH PROJECT - INNOVATION - ENGINEERING ENGLISH

After a call for tenders from the corporate sector (aeronautics, spacial, pharmaceutical sectors…) or scientific laboratories (IMT, LISBP, LAAS, IMFT, LPNO, CEMES, …), students have to solve a mathematical modelling problem, supervised by specialists in the concerned domain (physics, mechanics, biology, finance, reliability...).
The essential point is the confrontation with a complex problem, whose solution is not known, even by the supervisor. This is the typical situation faced in research or innovation in industry and is a key factor in learning both autonomy and collaborative teamwork.
Through targeted activities (presentations in context, written accounts of experience...) the students will develop a working knowledge of the specificities of scientific English and be able to adapt their language strategies to specialists and non-specialists.

HIGH DIMENSIONAL AND DEEP LEARNING (HDDL)

This course is dedicated to learning methods complementary to those studied in year 4 (kernel methods, boosting ..) and to deep learning methods for processing complex data such as signals or images. It also discusses methods for anomaly detection.
Kernel methods: Support Vector Machine and Support Vector Regression.
Aggregation with Boosting and Extreme Gradient Boosting.
Convolutional neural networks: convolutional layer, pooling, dropout, convolutional network architectures (ResNet, Inception), transfer learning and fine tuning, applications for image or signal classification.
Encoder-decoder, Variational auto-encoder, Generative adversarial networks
Functional decomposition on splines, Fourier or wavelets bases or Functional PCA: cubic splines, penalized least squares criterion, Fourier basis, wavelet bases, applications to nonparametric regression, linear estimators and nonlinear estimators by thresholding, links with the LASSO method. Functional PCA
Anomaly detection: Main algorithms (One Class SVM, Random Forest, Isolation Forest, Local Outlier Factor); Applications to anomaly detection in functional data.

ELECTIVE COURSES: ONE AMONG TWO

1. COMPUTER EXPERIMENTS AND EXPERIMENTAL DESIGN

Experimental Design
Linear covariance models, multiple interactions, mixed models.
Principle of randomized experiments and classical experiments design
Factorial, fractional designs
Examples with the SAS or JMP software

Computer Experiment
Introduction: computer experiments and metamodelling. Examples of applications
Two famous metamodels: chaos polynomials and Gaussian process regression (Kriging)
Simulation of unconditional / conditional Gaussian processes
Accounting for external knowledge and covariance kernel customization
Metamodel-based optimisation (Bayesian optimisation)
Design of computer experiments: focus on space-filling design
Global sensitivity analysis: focus on ANOVA decomposition (Sobol decomposition)
Industrial application: uncertainty quantification

2. COMPUTER EXPERIMENTS AND STOCHASTIC CALCULUS WITH APPLICATIONS TO PDE MODELING

Stochastic calculus with applications to PDE modeling
Continuous-time stochastic processes and martingales. Introduction to stopping-times.
Construction of the Brownian movement and the stochastic integral then derivation of Itô's formula. Solving a Dirichlet problem using the Brownian motion.
Introduction to stochastic differential equations (SDE) then derivation of the Fokker-Planck equations. Solving a parabolic equation using the solution of a SDE.

Computer Experiment
Introduction: computer experiments and metamodelling. Examples of applications
Two famous metamodels: chaos polynomials and Gaussian process regression (Kriging)
Simulation of unconditional / conditional Gaussian processes
Accounting for external knowledge and covariance kernel customization
Metamodel-based optimisation (Bayesian optimisation)
Design of computer experiments: focus on space-filling design
Global sensitivity analysis: focus on ANOVA decomposition (Sobol decomposition)
Industrial application: uncertainty quantification

ELECTIVE COURSES: THREE AMONG EIGHT

1. AI FRAMEWORKS

Introduction to the Spark Hadoop framework with PySpark.
Introduction to cloud computing with Google Cloud.
Introduction to container with docker
Introduction to natural language processing applying to text classification (text mining, vectorization, classification) and generation (recurrent neural network).
Recommendation system
Introduction to reinforcement learning.

2. ADVANCED MODELS AND NUMERICAL METHODS FOR FLUID MECHANICS

Compressible Flow Modelling:
- Models for compressible fluid dynamics. Discontinuous solutions and Rankine Hugoniot relationships. Riemann's problem. Application examples in the field of high speed flows.
- Finite Volume schemes of order 1 and 2. MUSCL method. Lab work
- High order numerical methods. Lab work

Two-phase flow modelling
- Models for two-phase flows. Application examples in the industrial and environmental fields.
- Numerical methods for two-phase flows.
- Lab work on the VOF method.

Modeling of turbulent flows
- Models for turbulent flows (k-epsilon, k-omega, L.E.S, wall laws) and application examples.
- Lab work on the modelling of turbulent boundary layers

3. ADVANCED MODELING IN COMPUTATIONAL STRUCTURAL MECHANICS

Numerical modeling of thin structures: Construction of a beam model from standard 3D solid elasticity; Variational formulation, connection with energy minimization and numerical resolution by the finite element method.
CAD-analysis relation: Fundamentals for describing geometries in CAD; Isogeometric analysis: spline finite elements; Application for the computation of beam models.

Model and computation of contact problem: Frictionless contact between elastic bodies, Signorini conditions; Introduction to variational inequalities; Numerical methods for contact problems (penalty, regularization, duality).
Image registration using the FE modeling: Digital image correlation; Data assimilation in experimental mechanics

4. IMAGE

Mathematical modeling of the main image acquisition devices.
Image restoration: Modeling, total variation, noise reduction, inverse problems (inpainting, deconvolution, compressed image restoration).
Image registration: Principle and Overview of variational models in registration and applications
Sparse representation in a dictionary of atoms: principle, l0 and l1 minimization, compressed sensing (case l0 and l1 with the RIP criterion), Orthogonal MatchingPursuit.
Image segmentation: shape optimization with the levelset method of the maximum flow. Modeling for image segmentation: Mumford-Shah model, Chan-Vese, Boykov-Jolly
Non-local methods: Discrete Universal Denoiser, NL-mean, non-local total variation.
Learning methods (not convex) dictionary learning, K-SVD.

5. MATHEMATICAL MODELS AND NUMERICAL METHODS FOR ACTUARIAL SCIENCES

Introduction and vocabulary for Actuarial risk management.
Definition and analysis of the Cramér- Lundberg model: compound Poisson processes, ruin, premium and risk reserve.
Some stochastic and machine learning algorithm applied to actuarial data.
Introduction to risk measures.

6. POISSON PROCESSES AND APPLICATION TO RELIABILITY THEORY AND ACTUARIAL SCIENCES

Introduction of the theoretical tools for: Homogeneous and inhomogeneous Poisson processes (definitions and basic properties); Inferential Statistics for homogeneous; Poisson processes (estimation and tests for the process intensity).
Simulation methods for Poisson processes.
Mini-Projects: Application and illustration on real and/or simulated data of different aspects of Poisson processes.

7. RELIABILITY AND LIFETIME ANALYSIS

Lifetime: Censored or truncated data, instantaneous hazard rate, ageing models, parametric and nonparametric (Kaplan-Meier and Nelson-Aalen) estimation, models with covariates (regression models of Cox and of Aalen), bayesian approach.
System structure and Reliability: reliability diagram, series system, parallel system, k/n or mixed system, structure function.
Reliability of repairable systems: Reliability and Availability, Markov models, failure intensity process, homogenous and non-homogeneous Poisson process models, renewal process, corrective and predictive maintenance, imperfect maintenance models, degradation models and maintenance policy.

8. VARIATIONAL DATA ASSIMILATION & MODEL LEARNING

Variational Data Assimilation.
Examples of inverse problems: least-squares, optimal command, parameter identification, data assimilation. Optimal control: ODE Linear-Quadratic case, maximum principle, Hamiltonian; PDE non-linear, adjoint equations, optimality system, Lagrangian.
Variational Data Assimilation. Cost function, optimisation, regularisations.
Linear case: links between VDA, BLUE, sequential methods, Bayesian view.
Model learning.
Learning a model (ODE or PDE) from large datasets; next derive a surrogate model.
Programming practical (Python-Fenics or FreeFem++): advection-diffusion models or Burgers equation.

Major in Math. Modeling & AI (2021-2022)