APPLIED MATHEMATICS FIELD_13 ECTS

Introducing

Description

Évaluation

L'évaluation des acquis d'apprentissage est réalisée en continu tout le long du semestre. En fonction des enseignements, elle peut prendre différentes formes : examen écrit, oral, compte-rendu, rapport écrit, évaluation par les pairs...

Practical info

Location(s)

Toulouse

Statistical Modelling

Introducing

Description

Programme (detailed contents):

-Nonparametric tests: empirical distribution function, goodness-of-fit test of Kolmogorov, tests of comparison of two populations, (Kolmogorov-Smirnov and Wilcoxon tests), normality tests (Kolmogorov and Shapiro-Wilk)
-Chi-square test of goodness-of-fit, of independence, of homogeneity
-Linear models: estimation of the parameters, confidence intervals, prediction intervals, Fisher test for a sub-model, variable selection
Linear regression, ANOVA, ANCOVA
-Generalized linear model: statistical inference, variable selection.
Logistic regression, loglinear model

Objectives

At the end of this module, the student will have understood and be able to explain (main concepts):

-The use of statistical tests for goodness-of-fit, independence, populations comparisons
-The characteristics of a linear model and a generalized linear model, and their use for statistical modelling

At the end of this module, the student should be able to:

-Choose a test procedure suited to a given problem
-Build nonparametric test procedures to compare two populations
-Build goodness-of-fit tests for a single distribution or a family of distributions
-Choose a linear model or a generalized linear model suited to a given problem
-Estimate the parameters in a linear model and a generalized linear model
-Use statistical tests to validate or invalidate hypotheses on these linear models and generalized linear models.
-Implement a variable selection strategy
-Perform a complete statistical analysis on a real data set using a linear model or a generalized linear model

Necessary prerequisites

Probability and Statistics (I2MIMT31)
Statistics (I3MIMT15)

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Optimisation

Introducing

Description

Program (detailed contents):

- Elements of convex analysis: convexity, lower semi-continuity, notion of subdifferential, elements of analysis for algorithmic purposes (Lipschitz gradient functions, strong convexity, conditioning)

- Optimality conditions (Karush Kuhn Tucker conditions, second order sufficient conditions)

- Lagrangian duality

- Algorithms for unconstrained differentiable optimization and link with ODEs: generalities on descent methods, gradient algorithms, Newton and quasi-Newton algorithms. Convergence study and convergence speed depending on the geometry of the functions to be minimized.

- Algorithms for constrained differentiable optimization: SQP, penalization methods, augmented Lagrangian.

- Convex optimization: how convexity can improve the convergence speed of algorithms.

- Inertial algorithms, Nesterov acceleration. Subgradient algorithms. Notion of proximal operator, Moreau regularization, proximal algorithms. Splitting methods: Forward Backward algorithm and Nesterov acceleration (FISTA). Study of convergence and convergence rate on the class of convex functions.

Objectives

At the end of this module, the student will have understood and be able to explain (main concepts):

* Existence and uniqueness conditions for solutions of an optimization problem (constrained differentiable optimization, non-differentiable convex optimization)

* Optimality conditions: Karush Kuhn Tucker points in the constrained differentiable case, optimality NHA in unconstrained convex optimization.

* The duality principle: Lagrangian duality, Fenchel-Rockafellar duality

* Gradient and Newton type algorithms, and their convergence results, classical algorithms of constrained optimization:

* The general principle of inertial algorithms: acceleration of gradient algorithms, generalization to the composite case.

The student will be able to:

- Interpret the behavior of an algorithm as a discretization of a dynamic system
- Identify classes of optimization problems and propose algorithms adapted to the geometry of the functions to be minimized.
- Implement and numerically calibrate these algorithms.

Necessary prerequisites

Basics of differential calculus and linear algebra.

3rd year MIC optimization course

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Practical info

Location(s)

Toulouse

PDE 2 (OPTIONAL TEACHING)

Introducing

Description

Program (detailed contents):

Modeling with PDE (nonlinear PDEs in one dimension space)
1. Ecology, Population Dynamics, Epidemiology (reaction-diffusion equations)
2. Traffic, Ecology (forest fire): nonlinear advection equations
3. Wave propagation phenomena (elliptic equations)

Elliptic equations : analysis and simulation

1. Variational formulation: weak derivatives, weak solutions, Sobolev spaces
2. Finite element methods (in one dimension space): Galerkin methods, P1 and P2 elements, convergence and error estimates, qualitative properties

Hyperbolic equations: analysis and simulation

1. Method of characteristics, weak solutions, formation of shocks in finite time, maximum principle
2. Finite volume methods (in one dimension space): centered and upwind schemes, discrete maximum principle, order 1 and 2 methods.

Objectives

At the end of this module, the student will have understood and be able to explain (main concepts):

¿ Main PDE models found in the literature in biology, ecology, physics and engineering.
¿ Notions of weak derivatives, weak solutions and variational formulations
¿ Finite elements and finite volume methods: formulation, implementation and convergence.

The student will be able to:

¿ Derive a variational formulation of an elliptic problem
¿ Implement finite elements method in 1d for elliptic equations.
¿ Solve nonlinear scalar hyperbolic equations with method of characteristic
¿ Implement finite volume method in 1d for hyperbolic equations.

Necessary prerequisites

Ordinary differential equations : modeling, existence of solutions, qualitative study, numerical simulation (convergence, stability, order)

Introduction to PDE (EDP1): linear equations, analytic resolution, finite difference methods

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Practical info

Location(s)

Toulouse

Advanced probability (OPTIONAL TEACHING)

Introducing

Description

Syllabus (detailed contents):
- Conditional expectation, filtration, martingale, submartingale and supermartingale, Doob¿s theorem, optional stopping theorem, convergence theorems, law of large numbers and central limit theorem for martingales. Parametric estimation through maximum likelihood estimation in Markovian models.
- Background on deterministic gradient descent, Introduction to Robbins-Monro algorithms and links with classical results (Law of Large Numbers), Robbins-Siegmund Lemma, Robbins-Monro Convergence Theorems, Applications (Two-Armed Bandit, quantile, quantization, Linear Regression in high dimension).

Objectives

At the end of this module, the student will have understood and be able to explain (main concepts):
- The notion of conditional expectation, the main properties of martingales and their classical use in modelling,
- Stochastic algorithms of Robbins-Monro type.

The student will be able to:
- To compute a conditional expectation, to show that a random process is a martingale, to use the various theorems (Doob, optional stopping and convergences), in particular for the maximum likelihood estimation.

- Build and study the convergence of stochastic optimization algorithms, apply these methods to different problems (quantile, quantization,¿)

The student will be able to:
- To compute a conditional expectation, to show that a random process is a martingale, to use the various theorems (Doob, optional stopping and convergences), in particular for the maximum likelihood estimation.

- Build and study the convergence of stochastic optimization algorithms, apply these methods to different problems (quantile, quantization,¿)

Simulate a random variable by different methods, use probabilistic, choose appropriate techniques for variance reduction and error estimation

Necessary prerequisites

Necessary knowledge:

A basic course on probabilities.

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